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FLOW WITH APPLICATION TO GAS FLOW LASERS

by

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KINETIC APPROACH TO RADIATIVE NONEQUILIBRIUM FLOW WITH APPLICATION TO GAS FLOW LASERS

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ABSTRACT

A kinetic approach to nonequilibrium flow of lasing gas is presented. The author introduces a new gain related to molecular speed (GMS) and develops an approximate method of solution. These treatments make it possible to exactly describe the interaction between radiative field, macroscopic flow and microscopic molecular motion. In the case of CO_2 gas flow lasers, the zero-order approximation solutions of this theory are already satisfactory in that they are valid for the whole pressure range. The results of the zero-order solutions agree well with numerical results, and are in accordance with those of the currently accepted rate-equation theory (RET) in the high pressure range. For zero flow speed, this theory leads to the well-known theory of non-flow gas lasers [11]. One of the present conclusions is specially worth noting, i.e., when low-pressure broadening constant $\iota < 0.2$, the rate-equation theory, although the line shape factor of the revised pressure effect was introduced [4,5], cannot correctly account for the effects of inhomogeneous broadening. For example, when $\eta=0.02$, $\bar{I}_R/\bar{I}_K$ are about 8 when $\xi=0$ and 20 when $\xi=1.0$, where ξ is the frequency shift parameter, $\bar{I}_R$ and $\bar{I}_K$ are the dimensionless radiative intensities of RET and this theory, respectively.

NOTATION

c is the speed of light

c_p is specific heat at constant pressure

F_i, F_i^0 speed distribution function of i-th energy-level particle, and distribution function of its equilibrium speed

f_i, f_i^0 $f_i = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_i dV_x dV_z$,

$f_i^0 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_i^0 dV_x dV_z$

f_ν distribution function of photon

G gain coefficient

G_T gain correlated to molecular speed

h Planck constant, or static entropy of gas flow

$\mathbf{J}$, J_s, J_t radiation intensity, saturated strength and penetrating radiation intensity

$\mathbf{k}_T$, k_r velocity of elastic collision, characteristic velocity of radiation

$\mathbf{k}_{ij}$, K_{ij} velocity of inelastic collision

$L_i (i = 1, 2, 3)$ length of optical cavity along the direction of the coordinate axis

$\mathbf{l}$, l_x l_y, l_z direction vector of light propagation, and three direction cosines

m molecular weight

n_i particle number density at i-th level

p gas pressure

$R_i = 1 - a_i - t_i$ reflective index a_i of mirror is absorption rate, while t_i is the penetrating rate

T, u gas flow temperature and flow velocity

$\mathbf{V}$. V_T particle velocity vector and thermal velocity vector

$r_i = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Gamma_i dV_x dV_z$ pumping velocity

ν, $\nu_{i,i-1}$ light frequency, transition frequency from i-th to (i-1)-th energy level

$\Delta\nu_D$, $\Delta\nu_N$ whole widths at half-peak value for inhomogeneous and homogeneous broadening type lines

ξ frequency shift parameter or transformation coordinate

η broadening parameter

λ_1, λ_2 intrinsic values

ρ gas flow density

δ constant

0 superscript 0 denotes motion along the gas flow direction, the initial position of laser oscillation

I. Introduction

In the study of radiation in equilibrium flow on the interaction between radiation and gas flow, emphasis is placed on the particle characteristics of radiation, but there is no consideration of wave motion structure [1,4]. Generally, the study can be divided into two categories: 1) $K_{cn} > K_r$, this is the situation for the study of the physical gas dynamics [1-3]; K_{cn} and K_r are, respectively, the characteristic velocity of intermolecular elastic collision and radiation transfer. In this situation, the molecular distribution of quantum energy levels is controlled by the collision process. The radiative transfer of the energy is a nonequivalent process. 2) $K_{cn} \leq K_r$, this is the situation with the existence of anti-Boltzmann distribution and laser emission as the feature, such as gas flow lasers [4,9]. To calculate the motion of laser medium gas and its radiation properties, generally the sets of simultaneous equations of fluid dynamics, radiation transfer, and velocity equations are solved simultaneously [4-6] (the set of velocity equations describes the variation of the Boltzmann constant of the energy level). For convenience, this is called the rate equation theory (RET). In RET, it is assumed that particles at different velocities at the same energy level can react with a monochromatic radiation field,

therefore, the inhomogeneous broadening effect cannot be correctly reflected. As pointed out by authors in reference [6], RET is not suitable to be used in the situation of medium to low gas pressure, but is suitable only for cases of high gas pressure.

From radiation theory, we know that only frequency-resonant molecules [7] (the doppler frequency for absorbing or induced emission molecules approaches the frequency of the radiation field) can have direct interaction with a monochromatic radiation field. Therefore, if the monochromatic radiation field is very intense and the gas pressure is low, that is, the homogeneous broadening is predominant, the frequency-resonant molecules within an energy level will be surplus (absorption situation) or insufficient (emission situation). That is, the velocity distribution function of the energy level will be protruding, or burning a hole [7]. It is not possible in RET to separate the molecules between the frequency-resonant molecules and those molecules unable to directly affect the radiation field because of excessive doppler frequency shift. It is necessary to explore the more rational model. The Lamb theory [8] and its extension make possible an ideal treatment of the inhomogeneous broadening effect in a situation in which gas properties do not vary with time and space. This article explores some aspects of kinematics of gas flow lasers in the situation when gas properties vary with the flow direction distance. The kinematic equations describe the variation of rate distribution function of energy level particles. In a study of kinematics, the interaction among gas particles in which the radiation field and the thermal and macroscopic motion can be well described. However, it is very difficult to solve the set of simultaneous equations relating to coupling between the flow field and the relaxation process, on the one hand, and radiative transfer, on the other. In this article, a new physical concept is introduced, concerning gain relating to thermal molecular velocity; moreover, an approximate

solution method is developed to overcome the difficulties. This concept is quite effective for use in gas flow lasers.

II. Model of Kinematics

1. Fundamental set of equations: regarding the quantum energy levels, the set of kinematic equations and the steady-state radiative transfer equation are, respectively, as follows:

$$
\begin{aligned}
\frac{\partial F_i}{\partial t} - (V + V_{T_i}) \operatorname{grad} F_i = \Gamma_i + k_T(F_i^0 - F_i) + \int K_{i+1,i} F_{i+1} \delta(V' - V_i) dV_{i+1} dV' \\
- \int K_{i,i-1} F_i dV_{i-1} - \int K_{i,i+1} F_i dV_{i+1} + \int K_{i-1,i} F_{i-1} \delta(V' - V_i) dV_{i-1} dV' \\
+ \int_0^\infty \int_0^{4\pi} f_\nu \phi_{i+1,i} (B_{i+1,i} \alpha_{i+1} F_{i+1} - B_{i,i+1} \alpha_i F_i) d\nu d\Omega \\
- \int_0^\infty \int_0^{4\pi} f_\nu \phi_{i,i-1} (B_{i,i-1} \alpha_i F_i - B_{i-1,i} \alpha_{i-1} F_{i-1}) d\nu d\Omega
\end{aligned}
\qquad (2.1)
$$

$$
c \boldsymbol{l} \operatorname{grad} f_\nu = f_\nu \sum_i \int \phi_{i,i-1} (B_{i,i-1} \alpha_i F_i - B_{i-1,i} \alpha_{i-1} F_{i-1}) d\nu' \qquad (2.2)
$$

$$
\phi_{i,i-1} = \frac{\Delta\nu_N}{2\pi} \left\{ \left[\nu - \nu_{i,i-1} \left(1 + \frac{1}{c} V_T^{\cdot} \cdot \boldsymbol{l} \right) \right]^2 + \left(\frac{\Delta\nu_N}{2} \right)^2 \right\}^{-1} \qquad (2.3)
$$

The set of equations (2.1) describes the relaxation process of the initial inequilibrium distribution toward the local-equilibrium Boltzmann-Maxwell distribution. In (2.1), the elastic collision integration was replaced by the B-G-K model [1]. The inelastic collision term is expressed phenomenologically. In the inelastic collision and the radiation terms, only a mono-quantum jump is considered; generally, a multiquantum jump can be neglected. Radiation pressure, spontaneous radiation and the contribution made by scattering are also neglected. $K_{i+1,i}$ indicates that the i-th energy level raises a particle to the given velocity category; the (i+1)-th energy level simultaneously loses the collision transfer velocity constant of a particle; $K_{i,i-1}$ indicates that the (i-1)-th energy level increases a particle; the i-th energy level given velocity category simultaneously loses the transfer velocity constant of a particle. $K_{i+1,i}$ and $K_{i,i+1}$ are related; this relationship can be derived from the principle of detailed balance; Γ_i is the pumping term, such as electron excitation, photoexcitation and

excitation by chemical reaction, and so on.

2. Gain relating to molecular velocity (GMS). When the characteristic radiation velocity $K_r(K_r \approx f_r\phi_{i,i-1}B_{i,i-1}a_i)$ is greater than the characteristic inelastic collision velocity $K_{ra}\left(K_{ra} \approx \int K_{i,i-1}dV_{i-1}\right)$ and is comparable with the elastic collision velocity k_T, in the energy-level spectral line shape, only frequency-resonant molecules (the molecules with consistent doppler frequency of absorption or induced emission, and the frequency of the monochromatic radiation field) can have a direct function with the monochromatic radiation field. In the spectral lines, the doppler frequency shift of other particles is overlarge, so it is unable to be directly related to the radiation field. Therefore, for the local deformation of energy-level spectral lines, the radiative transfer can be in competition with elastic collision transfer; the energy-level distribution function can possibly have a protruding or burning a hole in local places [7]. To describe this physical process, we introduce the gain G_{T_i}(GMS), relating to molecular velocity. The definition of G_{T_i} is as follows:

$$G_{T_i} = \frac{2}{c\pi\Delta\nu_N}\left(B_{i,i-1}a_iF_i - B_{i-1,i}a_{i-1}F_{i-1}\right) \qquad (2.4)$$

$G_{T_i}\phi_{i,i-1}$ indicates the gain coefficient of the gap for unit molecular velocity and the unit three-dimensional angle. Integrate the G_{T_i} versus the apparent frequency ν' of the molecule from -infinity to infinity to obtain the homogeneous broadening gain coefficient G_h in the conventional sense

$$G_{h,i} = \int G_{T_i}d\nu' \qquad (2.5)$$

As is the case in gas kinematics, approximating G_h can simplify the problem [1,2], approximating G_{T_i} can possibly simplify the problems relating to thermal molecular motion.

3. Approximate solution method. G_{T_i} is the function of the molecular apparent frequency ν', therefore in Eq. (2.1), G_{T_i} can be removed outside the signs of the double integral of frequency ν and the solid angle Ω ; if the G_{T_i} is considered as a function

of f_M and $(F_i^0 - F_i)$, then from Eq. (2.1) the approximate solution of G_{Ti} can be obtained. On the other hand, we solve the following double-parameter perturbation solution of Eq. (2.1):

$$F_i = \sum_{j=0} \sum_{k=0} \left(\frac{\varkappa}{L_1 k_T}\right)^j \left(\frac{\varkappa}{L_1 k_r}\right)^k F_i^{jk} \qquad (2.6)$$

Obviously, F_i^{00} is the Maxwell distribution function; F_i^{10} is the Chapman-Enskog solution. By substituting the approximate solution of G_{Ti} and the perturbation solution F_i in Eq. (2.2), the solution of the radiative transfer equation (2.2) can be obtained. By using the solution of Eq. (2.2), a simultaneous solution of the equation of macroscopic motion (the moment of the kinematics equation), the flow field variate p, T and the flow velocity can be obtained.

By utilizing the above-mentioned concept and methods, the following problem can be handled: 1) the situation of weak radiation, 2) the situation in which the time and space variation of the velocity distribution function is secondary, and 3) the situation with high radiation intensity and discrete frequency with a finite number of discretenesses. For the situation of CO_2 gas flow lasers, the zero-level solution is better than the results of conventional rate equation theory (RET) [4-6.

III. Gas Flow Lasers

In CO_2 gas flow lasers, the light beam direction is perpendicular to the flow direction (refer to Fig. 1); the flow in the optical cavity is approximately one-dimensional in nature. The effect of viscosity can be neglected and the pumping function is uniform and continuous. The molecular relaxation of a CO_2 gas mixture is consistent with reference [5]. The relaxation model is composed of five energy-level groups (refer to Fig. 2); therefore, five velocity distribution functions are required. That is, $F_i (i = 0, 1, 2, 3)$ and $F_{0'}$, 0 indicates the ground state

of CO_2 vibrations; 1 and 2 indicate, respectively, the CO_2 symmetric-bending and nonsymmetric vibrational models. 0′ and 3 are the ground state and oscillation model of the diatomic molecule. The P-branch laser jump of CO_2 occurs between the vibrational-rotational energy level (0, 0, 1; j) and (1, 0, 0; j+1); j is the number of rotational quantum. Overlapping each vibration energy level, a series of rotational energy levels are not shown in Fig. 3; however, the effect of rotational energy levels has been absorbed in factors α_1 and α_2.

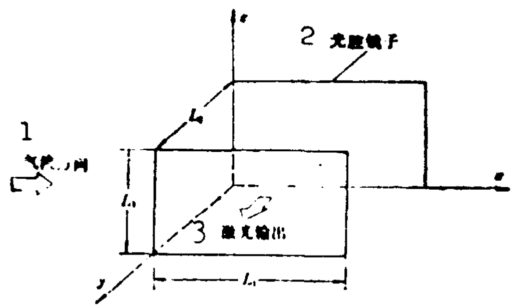

Fig. 1. Optical cavity and coordinate system
KEY: 1 - direction of gas flow 2 - mirror of optical cavity 3 - laser output

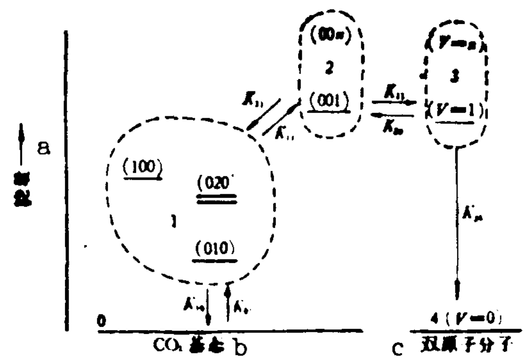

Fig. 2. Relaxation model of molecular system
KEY: a - energy quantity
b - CO_2 ground state
c - diatomic molecule

The laser beam is parallel to the y-axis, $l \cdot V_T = V_{T_y}$. The x and z components of the thermal molecular motion do not affect the radiation field; therefore, we can integrate Eq. (2.1) with respect to V_{T_x} and V_{T_z} to obtain the kinematics equation of

$$u\frac{\partial f_1}{\partial x} = \gamma_1 + k_T(f_1^0 - f_1) + k_{21}f_2 - k_{10}f_1 + f_\nu\phi_{21}(B_{21}\alpha_2 f_2 - B_{12}\alpha_1 f_1) \quad (3.1)$$

$$u\frac{\partial f_2}{\partial x} = \gamma_2 + k_T(f_2^0 - f_2) + k_{32}f_3 - k_{23}f_2 - k_{21}f_2 - f_\nu\phi_{21}(B_{21}\alpha_2 f_2 - B_{12}\alpha_1 f_1) \quad (3.2)$$

$$u\frac{\partial f_3}{\partial x} = \gamma_3 + k_T(f_3^0 - f_3) - k_{32}f_3 + k_{23}f_2 \quad (3.3)$$

$$\int_{-\infty}^{\infty}(f_0 + f_1 + f_2)\,dV_{T_y} = \text{const} \quad \int_{-\infty}^{\infty}(f_{0'} + f_3)\,dV_{T_y} = \text{const} \quad (3.4)$$

$$\frac{\partial f_\nu}{\partial y} = \frac{f_\nu}{c}\int_{-\infty}^{\infty}\phi_{21}(B_{21}\alpha_2 f_2 - B_{12}\alpha_1 f_1)\,d\nu' \quad (3.5)$$

In deriving Eqs. (3.1)-(3.3), the following approximations were

adopted:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V_{Ti}\,\mathrm{grad}\,F_i\,dV_{xi}\,dV_{zi} = \frac{\partial}{\partial x}\left(\iint V_{T_{xi}} F_i\,dV_{xi}\,dV_{zi}\right) \tag{3.6}$$

$$+\frac{\partial}{\partial y}\left(\iint V_{T_{yi}} F_i\,dV_{xi}\,dV_{zi}\right) + \frac{\partial}{\partial z}\left(\iint V_{T_{zi}} F_i\,dV_{xi}\,dV_{zi}\right) \tag{3.7}$$

$$\approx V_{T_{yi}}\frac{\partial f_i}{\partial y} \approx 0$$

$$\int K_{i+1,i}F_{i+1}\delta(V' - V_i)dV_{i+1}dV' \approx k_{i+1,i}F_{i+1} \tag{3.8}$$

$$\int K_{i-1,i}F_{i-1}\delta(V' - V_i)dV_{i-1}dV' \approx k_{i-1,i}F_{i-1} \ll \int K_{i,i-1}F_i dV_{i-1}$$

$$\approx k_{i,i-1}F_i$$

It was expressed in Eq. (3.6) that the variation of f_i along the y-axis was neglected. Eq. (3.7) is another phenomenological expression of the inelastic collision term; this actually is consistent with the phenomenological expression in the last section. As expressed in Eq. (3.8), the transition velocity from the i-th to the (i-1)-th energy level is greater than the reverse process, that is, the transition velocity from the (i-1)-th to the i-th energy level. However, $k_{32}F_3$ can be comparable to $k_{23}F_2$. This is because the transfer between the diatomic molecule vibrational mode and CO_2 asymmetric vibrational mode is near-resonant.

On a mirror surface, radiation satisfies the following boundary conditions:

$$y = 0,\ J_0^+ = R_1 J_0^-;\ y = L_2,\ J_{L_2}^- = R_2 J_{L_2}^+ \tag{3.9}$$

In the equation, J^+ and J^- are, respectively, the radiation intensities of positive- and negative-direction propagation along the y-axis: $J = J^+ + J^-$, $J = ch\nu f_n$.

IV. Solution Procedure

In the case of monochromatic radiation, for computational convenient G_T is rewritten as

$$G_T = \frac{1}{c}\phi_{21}(B_{21}a_2f_2 - B_{12}a_1f_1) \tag{4.1}$$

By utilizing Eqs. (3.1), (3.2), and (4.2), we can derive the fact that the control equation of G_T is:

$$\begin{aligned} s_0\frac{\partial G_T}{\partial \xi} &+ \left[s_0(k_{21} + s_2k_{23} + s_1k_{10}) + \frac{J}{h\nu}\right]G_T \\ &- s_2\gamma_2 - s_1\gamma_1 + \sum_{\lambda=1}^{2} s_i k_T(f_i^0 - f_i) + s_2k_{32}f_3 - s_1(k_{21} + s_2k_{23} - s_2k_{10})f_b \end{aligned} \tag{4.2}$$

in the equation, $\xi = \int \frac{1}{u}\,dx$

$$\begin{gathered} f_b = f_1 + f_2,\ f_2 = s_1f_b + s_0G_T,\ f_1 = s_2f_b - s_0G_T \\ s_0 = c[(B_{21}a_2 + B_{12}a_1)\phi_{21}]^{-1},\ s_1 = B_{12}a_1(B_{21}a_2 + B_{12}a_1)^{-1},\ s_1 + s_2 = 1 \end{gathered} \tag{4.3}$$

According to Eq. (2.4), the dual parameters of f_i can be expanded into

$$\begin{gathered} f_i = \sum_{j=0}\sum_{k=0}\left(\frac{u}{L_1k_T}\right)^j\left(\frac{u}{L_1k_r}\right)^k f_i^{jk}\ (i = 1, 2) \\ f_3 = \sum_{j=0}\left(\frac{u}{L_1k_T}\right)^j f_3^j \end{gathered} \tag{4.4}$$

By substituting Eq. (4.4) into Eqs. (3.1)-(3.3) and into Eq. (4.1), we obtain the result that the zero-level solutions of f_i and G_T are

$$f_1^{00} - f_1^0 = -(f_2^{00} - f_2^0) \approx \frac{J_\nu\phi_{21}}{k_T}(B_{21}a_2f_2^{00} - B_{12}a_1f_1^{00}) \approx 0 \tag{4.5}$$

$$G_T^{00} = \frac{\phi_{21}}{c}(B_{21}a_2f_2^{00} - B_{12}a_1f_1^{00}) \approx \frac{\phi_{21}}{c}(B_{21}a_2f_2^0 - B_{12}a_1f_1^0) \tag{4.6}$$

f_i is the Maxwell distribution, that is,

$$\begin{gathered} f_i^{00} \approx f_i^0 = n_iM(T)\ (i = 1, 2, 3) \\ M(T) = \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}}\exp\left(-\frac{m}{2kT}V_{T_y}^2\right) \end{gathered} \tag{4.7}$$

By integrating the molecular apparent frequency ν' with respect to G_T^{00}, we obtain the generally adopted gain coefficient [4,5]. From Eq. (4.2), we obtain a more precise approximate solution G_T^0 for G_T than G_T^{00}. G_T^0 is called the semi-order solution. Thus, first we obtain n_i, that is, the specifically expressed equation

of f_i^0. By processing Eqs. (3.1) and (3.2), we obtain

$$\frac{\partial^2 f_0^0}{\partial \xi^2} + A_1 \frac{\partial f_0^0}{\partial \xi} + A_2 f_0^0 = s_0(k_{10} - k_{23}) \frac{\partial G_T^0}{\partial \xi} + A_3 G_T^0 + A_4$$

$$\frac{\partial^2 f_3^0}{\partial \xi^2} + B_1 \frac{\partial f_3^0}{\partial \xi} + B_2 f_3^0 = s_0 k_{23} \frac{\partial G_T^0}{\partial \xi} + B_3 G_T^0 + B_4$$

(4.8)

In the equations, A_i and B_i are functions of k_{ij} and γ_i; k_{ij} is also a function of p and T. As expressed in experimentation and analysis, then $k_{ij} \propto p T^{\beta}$ $(0 < \beta < 1)$[13,5,10]. Therefore, solving for Eq. (4.8) should be done simultaneously with the set of gas macroscopic motion equations. To obtain the approximate solution of Eq. (4.8), the following mathematical transformation is introduced:

$$\zeta = \int^{\xi} \sqrt{\mu}\, d\xi = \int^{x} \frac{\sqrt{\mu}}{u} dx, \quad \mu = s_0 k_{32} k_{10}$$

$$\frac{\partial}{\partial \xi} = \sqrt{\mu} \frac{\partial}{\partial \zeta}, \quad \frac{\partial^2}{\partial \xi^2} = \mu \frac{\partial^2}{\partial \zeta^2} + \frac{\partial \sqrt{\mu}}{\partial \xi} \frac{\partial}{\partial \zeta}$$

(4.9)

By substituting Eq. (4.9) into Eq. (4.8) and neglecting the small value term $\frac{u}{L_1 \sqrt{\mu}}$, we obtain

$$\frac{\partial^2 f_0^0}{\partial \zeta^2} + \frac{k_{12} + s_1 k_{23} + s_2 k_{10}}{\sqrt{\mu}} \frac{\partial f_0^0}{\partial \zeta} + f_0^0 = \frac{s_0(k_{10} - k_{23})}{\sqrt{\mu}} \frac{\partial G_T^0}{\partial \zeta} + \frac{s_0}{s_2} G_T^0 + \frac{1}{s_2 k_{10}} \sum_{i=1}^{3} \gamma_i$$

$$\frac{\partial^2 f_3^0}{\partial \zeta^2} + \frac{k_{32} + s_1 k_{23} + s_2 k_{10}}{\sqrt{\mu}} \frac{\partial f_3^0}{\partial \zeta} + f_3^0 = \frac{s_0 k_{23}}{\sqrt{\mu}} \frac{\partial G_T^0}{\partial \zeta} + \frac{s_0 k_{23}}{s_2 k_{32}} G_T^0$$

$$+ \frac{s_1 k_{23}}{\sqrt{\mu}} \sum_{i=1}^{3} \gamma_i + \frac{\gamma_3}{k_{32}}$$

(4.10)

In Eq. (4.10), the coefficient of the first order partial derivative and the intrinsic values (minus signs) satisfy the following relationship

$$\frac{k_{32}+s_1k_{13}+s_2k_{10}}{\sqrt{\mu}} - \sqrt{\frac{k_{32}}{s_2k_{10}}} + \left(1+\frac{s_1k_{13}}{s_2k_{10}}\right)\sqrt{\frac{s_2k_{10}}{k_{32}}} \sim \text{const}$$

$$\lambda_1 \approx \sqrt{\frac{k_{32}}{s_2k_{10}}},\quad \lambda_2 \sim \sqrt{\frac{s_2k_{10}}{k_{32}}} \tag{4.11}$$

The intrinsic value (not related to the ζ-approximation) is equal to a constant. In the following, we discuss the solution $\lambda_1 \neq \lambda_2$; the solution of $\lambda_1 = \lambda_2$ can be extrapolated in our discussion.

$$\begin{aligned} f_1^0 = \sum_{i=1(j\neq i,\, j=1,2)}^{2} \frac{e^{-\lambda_i\zeta}}{\lambda_j-\lambda_i}\Bigg\{ & \frac{\gamma_1^0+\gamma_2^0}{\sqrt{\mu^0}} + \left[n_3^0\sqrt{\frac{k_{32}}{s_2k_{10}}} + \left(\lambda_j - \frac{s_1k_{13}}{\sqrt{\mu}} - \sqrt{\frac{s_2k_{10}}{k_{32}}}\right)n_2^0\right] M(T) \\ & + \frac{s_0(k_{10}-k_{23})}{\sqrt{\mu}}\, G_T^0\Big|_{\zeta=0} + \int_0^\zeta e^{\lambda_i\zeta}\Bigg[\frac{1}{s_2k_{10}}\sum_i^3 \gamma_i + \frac{s_0(k_{10}-k_{23})}{\sqrt{\mu}}\frac{\partial G_T^0}{\partial \zeta} \\ & + \frac{s_0}{s_2}\, G_T^0\Bigg] d\zeta\Bigg\} \end{aligned}$$

$$\begin{aligned} f_2^0 = \sum_{i=1(j\neq i,\, j=1,2)}^{2} \frac{e^{-\lambda_i\zeta}}{\lambda_j-\lambda_i}\Bigg\{ & \frac{\gamma_3^0}{\sqrt{\mu^0}} + \left[\left(\lambda_j - \sqrt{\frac{k_{32}}{s_2k_{10}}}\right)n_3^0 + \frac{s_1k_{23}}{\sqrt{\mu}}\, n_2^0\right] M(T) \\ & + \frac{s_0k_{23}}{\sqrt{\mu}}\, G_T^0\Big|_{\zeta=0} + \int_0^\zeta e^{\lambda_i\zeta}\Bigg[\frac{s_1k_{23}}{\mu}\sum_i^3 \gamma_i + \frac{\gamma_3}{k_{32}} + \frac{s_0k_{23}}{\sqrt{\mu}}\frac{\partial G_T^0}{\partial \zeta} \\ & + \frac{s_0k_{23}}{s_2k_{32}}\, G_T^0\Bigg] d\zeta\Bigg\} \end{aligned} \tag{4.12}$$

The relationship between G_T^0 and ζ : by integrating Eq. (3.5) with respect to y and utilizing the radiation boundary condition (3.9), we can derive

$$\frac{1}{L_2}\int_0^{L_2}\int_{-\infty}^{\infty} G_T^0 dv' dy = -\frac{1}{2}\ln R_1R_2 \tag{4.13}$$

The reflectivity (of the mirror) R_i (i = 1, 2) does not vary with x [4-6], therefore we can generally assume: $\ln R_1R_2 = e^{\delta\zeta}\ln R_1^0R_2^0$, here δ is a constant or is equal to 0. In the kinematics equation, the thermal velocity V_T, is not related to the space coordinates. Therefore, finally we have

$$G_T^0 \propto \ln R_1R_2 = e^{\delta\zeta}\ln R_1^0R_2^0 \tag{4.14}$$

Besides, because of $\lambda_i = O(1)$, $\frac{1}{\lambda_i}\frac{\partial F}{\partial \zeta} = O\left(\frac{\kappa F}{\lambda_i L\sqrt{\mu}}\right) \ll O(F)$, therefore we have

$$\int_0^\zeta e^{\lambda_i \zeta} F d\zeta = \left(\frac{F}{\lambda_i} e^{\lambda_i \zeta}\right)\Big|_0^\zeta - \int_0^\zeta \frac{e^{\lambda_i \zeta}}{\lambda_i} \frac{\partial F}{\partial \zeta} d\zeta \approx \frac{1}{\lambda_i}[e^{\lambda_i \zeta} F(\zeta) - F(0)] \tag{4.15}$$

By utilizing Eqs. (4.14) and (4.15), integrate Eq. (4.12) and thus we obtain

$$\begin{aligned}
f_b^0 &= f_{bp} + s_0 \omega_b G_T^0 + \sum_{i=1(j\neq i, j=1,2)}^{2} \frac{e^{-\lambda_i \zeta}}{\lambda_j - \lambda_i} \Bigg\{ s_0 G_T^0 e^{-\delta\zeta} \left[\frac{k_{10} - k_{23}}{\sqrt{\mu}} - (\lambda_i + \delta)\omega_b^0 \right] \\
&\quad + \left[\frac{\gamma_1' + \gamma_2'}{\sqrt{\mu^0}} - \lambda_j n_{bp}^0 + n_3^0 \sqrt{\frac{k_{32}}{s_0 k_{23}}} + \left(\lambda_j - \frac{s_1 k_{23}}{\sqrt{\mu_1}} - \sqrt{\frac{s_0 k_{10}}{k_{32}}} \right) n_b^0 \right] M(T) \Bigg\} \\
f_3^0 &= f_{3p} + s_0 \omega_3 G_T^0 + \sum_{i=1(j\neq i, j=1,2)}^{2} \frac{e^{-\lambda_i \zeta}}{\lambda_j - \lambda_i} \Bigg\{ s_0 G_T^0 e^{-\delta\zeta} \left[\frac{k_{23}}{\sqrt{\mu}} - (\lambda_i + \delta)\omega_3^0 \right] \\
&\quad + \left[\frac{\gamma_3'}{\sqrt{\mu^0}} - \lambda_j n_{3p}^0 + \left(\lambda_j - \sqrt{\frac{k_{32}}{s_2 k_{10}}} \right) n_3^0 + \frac{s_1 k_{23}}{\sqrt{\mu}} n_b^0 \right] M(T) \Bigg\}
\end{aligned} \tag{4.16}$$

In the equation

$$\gamma_i = \gamma_i' M(T)$$

$$\begin{aligned}
f_{bp} &= \frac{1}{s_0 k_{10}} \sum_i^3 \gamma_i \qquad f_{3p} = \frac{s_1 k_{23}}{\mu} \sum_i^3 \gamma_i + \frac{\gamma_3}{k_{32}} \\
\omega_b &= \frac{1}{(\lambda_1 + \delta)(\lambda_2 + \delta)} \left[\frac{1}{s_2} + \frac{(k_{10} - k_{23})\delta}{\sqrt{\mu}} \right], \\
\omega_3 &= \frac{1}{(\lambda_1 + \delta)(\lambda_2 + \delta)} \left(\frac{k_{23}\delta}{\sqrt{\mu}} + \frac{k_{23}}{s_2 k_{32}} \right)
\end{aligned} \tag{4.17}$$

By substituting Eqs. (4.14) and (4.16) in Eq. (4.2), the semiorder G_T^0 of G_T

$$G_T^0 = G_{on} M(T) \left(\bar{I} + \frac{2}{\pi \Delta \nu_N \phi_{21}} \right)^{-1} \tag{4.18}$$

In the equation, the specific expression $\bar{I} = \frac{J}{J_s}$, J_S and G_{on} can be referred to (5.3)

Flow field solution: by utilizing the solution f_i^0 and G_T^0, as well as Eqs. (4.16) and (4.18), the one-dimensional nonadiabatic flow equation can be obtained; this is the solution of the moment equation of the equations (3.1) through (3.4):

$$\rho u A = \text{const}$$

$$\rho u^2 + p = \text{const}$$

$$h = h^0 + \frac{u^2 - u^{0 2}}{2} - \int_0^\zeta u^2 \frac{\partial \ln A}{\partial \zeta} d\zeta = \int_0^\zeta \frac{Q}{\rho} d\zeta - \int_0^\zeta \int_{-\infty}^{\infty} \frac{J G_T^0}{\rho \sqrt{\mu}} dV_{T_y} d\zeta$$

$$h = C_p T + \sum_i^{s} \frac{\varepsilon_i n_i}{\rho}, \quad p = \rho \frac{k}{m} T \tag{4.19}$$

In the equation ϵ_i is energy of the energy vibrational level. Here we obtain the zero-level approximate solution of the set of kinematics and radiation transfer simultaneous equation (3.1) through (3.5) for the CO_2 gas flow laser. Given $\ln R \cdot F_2$, γ_i and the initial conditions, we can determine the twelve unknowns ρ, u, p, T, h, J, f_i^0 (i = 0, 1, 2, 3), f_b^0 and G_T^0 from 12 relationship equations (3.4), (4.3), (4.5), (4.12) and (4.18). In the following, some useful relationships are derived.

5. Gain, Intensity and Power

1. Relationship between gain and intensity: by integrating the solution (4.18) of G_T^0 with respect to ν' (apparent frequency of molecules), we obtain the relationship between gain and intensity, namely

$$G = \int_{-\infty}^{\infty} G_T^0 d\nu' = \frac{G_{0s}\Phi(\xi, \eta, \bar{I})}{1 + \bar{I}} \tag{5.1}$$

in the equation

$$\Phi(\xi, \eta, \bar{I}) = \frac{\eta^2(1+\bar{I})}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-t^2}}{\eta^2(1+\bar{I}) + (\xi - t)^2} dt \tag{5.2}$$

$$\xi = \frac{2(\nu - \nu_0)}{\Delta\nu_D}\sqrt{\ln 2}, \quad t = \frac{2(\nu' - \nu_0)}{\Delta\nu_D}\sqrt{\ln 2}, \quad \nu' = \nu_0\left(1 + \frac{1}{c} V_{T_y}\right)$$

$$\eta = \frac{\Delta\nu_N}{\Delta\nu_D}\sqrt{\ln 2}$$

$$\frac{2}{\pi\Delta\nu_N} \cdot \frac{J_s}{h\nu\sqrt{\mu}} = \frac{c s_2}{B_{21}a_2}\Bigg\{\frac{k_{21} + s_2 k_{23} + s_1 k_{10}}{\sqrt{\mu}} + \delta - \omega_3\sqrt{\frac{s_2 k_{32}}{k_{10}}} + \frac{s_1 H}{\sqrt{\mu}}\omega_b$$
$$- \sum_{i=1(j\neq i/j=1,2)}^{2} \frac{s_0^0 e^{-(\lambda_i+\delta)\zeta}}{s_0(\lambda_j - \lambda_i)}\Bigg(\Bigg[\frac{k_{23}}{\sqrt{\mu}} - (\lambda_j + \delta)\omega_3^0\Bigg]\sqrt{\frac{s_2 k_{32}}{k_{10}}}$$
$$- \frac{s_1 H}{\sqrt{\mu}}\Bigg[\frac{k_{10} - k_{23}}{\sqrt{\mu}} - (\lambda_j + \delta)\omega_b^0\Bigg]\Bigg)\Bigg\}$$

$$\frac{G_{0s} J_s}{h\nu\sqrt{\mu}} = \frac{1}{\sqrt{\mu}}\left(\gamma_2' + \gamma_3' - \frac{s_1 k_{21}}{s_2 k_{10}}\sum_i^3 \gamma_i'\right) + \sum_{i=1(j\neq i/j=1,2)}^{2} \frac{e^{-\lambda_i\zeta}}{\lambda_j - \lambda_i}\Bigg\{\left(\frac{\gamma_3'}{\sqrt{\mu^0}} - \lambda_j n_{3p}^0\right)$$
$$\cdot \sqrt{\frac{s_2 k_{32}}{k_{10}}} - \frac{s_1 H}{\sqrt{\mu}}(\gamma_1' + \gamma_2' - \lambda_j n_{bp}^0) + \Bigg[\frac{s_1 k_{23}}{k_{10}} - \frac{s_1 H}{\sqrt{\mu}}\Bigg(\lambda_j - \frac{s_1 k_{23}}{\sqrt{\mu}}$$

$$- \sqrt{\frac{s_2 k_{10}}{k_{32}}}\Big)\Big] n_i^0 + \left[\left(\lambda_i - \sqrt{\frac{k_{32}}{s_2 k_{10}}}\right)\sqrt{\frac{s_2 k_{32}}{k_{10}}} - \frac{s_1 H}{s_2 k_{10}}\right] n_3^0 \Big\} \tag{5.3}$$

$$H = k_{21} + s_2 k_{23} - s_2 k_{10}$$

when $\nu=\nu_0$ (that is, the optical frequency and the linear center frequency are consistent), Eq. (5.2) can be simplified as

$$G = \frac{G_{0n}\,\eta\sqrt{\pi}}{\sqrt{1+\bar{I}}}\exp[\eta^2(1+\bar{I})]\cdot[1+\mathrm{erf}\,(\eta\sqrt{1+\bar{I}})] \tag{5.4}$$

Eqs. (5.2) and (5.4) are adaptable when gain is equal to loss.

2. Intensity: According to the definition and relationship (3.5), the penetrating intensity is derived as

$$J_t = t_1 J_0^- + t_2 J_{L_1}^+ = \frac{(t_1\sqrt{R_2} + t_2\sqrt{R_1})L_2 J_s}{(\sqrt{R_1}+\sqrt{R_2})(1-\sqrt{R_1 R_2})}\,\frac{G_{0n}\bar{I}\phi(\xi,\eta,\bar{I})}{1+\bar{I}} \tag{5.5}$$

If at one end there is a totally reflective lens without any loss, that is, $R_2=1$, and on the other end, there is the penetrating output lens, for the situation of mainly $\nu=\nu_0$, as well as homogeneous and inhomogeneous broadening, we obtain, respectively, the following:

$$J_t = \frac{t_1 J_s}{a_1+t_1}\left(G_{0n}L_2 + \frac{1}{2}\ln R_1\right) \tag{5.6}$$

$$J_t = \frac{t_1 J_s}{a_1+t_1}\left(\pi\eta^2 L_2\frac{G_{0n}^2}{G} + \frac{1}{2}\ln R_1\right) \tag{5.7}$$

When u=0, and p and T are constant, the above formulas are simplified into a well-known relationship [11] of gas (not flowing) lasers; however, we should pay attention to the distinction between them. Here, $G_{0n} = G_{0n}(\zeta)$.

3. Power: power can be obtained by integrating J_t with respect x. For the situation $\nu=\nu_0$ and one end output, we obtain the output power P as follows from Eq. (5.5):

$$P = \frac{V_D}{L_1}\int_0^{L_1}\frac{t_1 u J_s G_{0n}}{(a_1+t_1)\sqrt{\mu}}\,\frac{\bar{I}\eta\sqrt{\pi}\exp[\eta^2(1+\bar{I})]}{\sqrt{1+\bar{I}}}\,[1-\mathrm{erf}\,(\eta\sqrt{1+\bar{I}})]\,d\zeta \tag{5.8}$$

In the equation, $V_D=L_1L_2L_3$ with mainly homogeneous and inhomogeneous broadening, Eq. (5.9) can be converted into

$$P = \frac{t_1}{a_1+t_1}\,\frac{V_D I_s^0}{L_1}\left(G_{0n}^0 L_2 + \frac{1}{2}\ln R_1^0\right) \tag{5.9}$$

$$P = \frac{t_1}{a_1 + t_1} \frac{V_D I_i^*}{L_2} \left(\frac{G_0^{*2} L_2}{G_0^*} + \frac{1}{2} \ln R_1^* \right) \tag{5.10}$$

In the equation

$$I_i^* = \frac{1}{L_1} \int_0^{\zeta_1} \frac{u J_i e^{\delta\zeta}}{\sqrt{\mu}} d\zeta, \quad G_{0n}^* = \int_0^{\zeta_1} \frac{u J_i G_{0n}}{\sqrt{\mu}} d\zeta \left[\int_0^{\zeta_1} \frac{u J_i e^{\delta\zeta}}{\sqrt{\mu}} d\zeta \right]^{-1}$$

$$\frac{G_0^{*2}}{G^*} = \int_0^{\zeta_1} \frac{u G_{0n}^2 J_i \eta^2 \pi}{\sqrt{\mu}} d\zeta \left[\int_0^{\zeta_1} \frac{u J_i e^{\delta\zeta}}{\sqrt{\mu}} d\zeta \right]^{-1}$$

PL_2/V_D is the penetrating radiation intensity by averaging the output length area; Eqs. (5.9) and (5.10) are consistent with the corresponding power relationship) [11] of the gas (not flowing) laser. However, here I_i^*、 G_{0n} and G_0^{*2}/G^* are the average quantities in the flowing direction. By utilizing Eqs. (4.15) and (5.3), we can derive the approximate explicit expression equation as I_i^*、 G_{0n} and G^{*2}/G^*.

6. Analysis and Discussion

1. Comparison with the exact numerical solution: refer to Table 1 for parameters used in the exact numerical solution; the corresponding broadening parameter η is equal to 2.5; this is the situation mainly of homogeneous broadening. The exact results are obtained from the simultaneous solutions of the fluid dynamic equation and the rate equation. In the calculations, the condition is used in which gain is equal to loss. The results of the approximate solution and the exact numerical solution match quite closely (refer to Figs. 4 through 6). We should point out that the double integration item in the energy equation solution (4.18) can be integrated by the same method as power integration.

2. Comparison with the rate equation theory (RET): usually simultaneous solutions of RET [4,5] are obtained for the set of fluid mechanics and the rate equations. For comparison, in the following we briefly derive the results of RET corresponding to Eq. (5.1).

The set of rate equations of the CO_2 laser gas mixture are:

$$\left.\begin{aligned} u\frac{dn_1}{dx} &= \gamma_1 + k_{21}n_2 - (k_{10} + k_{12})n_1 + h\nu GJ \\ u\frac{dn_2}{dx} &= \gamma_2 + k_{32}n_3 - (k_{23} + k_{21})n_2 + k_{12}n_1 - h\nu GJ \\ u\frac{dn_3}{dx} &= \gamma_3 - k_{32}n_3 + k_{23}n_2 \end{aligned}\right\} \tag{6.1}$$

TABLE 1. Calculation Conditions and Parameters at Inlet of Optical Cavity

$s_1 + s_2 = 1,\ s_1 = 0.96$	$N_3^0 = 7.73 \times 10^{16}$ (粒子/厘米³) e
$\frac{s_t}{s_0} = \frac{718}{NT}$ (厘米²) i	$N_2^0 = 2.30 \times 10^{16}$ (粒子/厘米³) e
$g = \frac{-1}{2L_1}\ln R_1 = 5 \times 10^{-3}$ (厘米⁻¹) j	$N_0^0 = 3.83 \times 10^{17}$
A = 常数 a	$N^0 = 9.66 \times 10^{17}$
$e_1 = 2.8 \times 10^{-13}$ (尔格/粒子) b	$\rho^0 = 4.0 \times 10^4$ (达因/厘米²) f
$e_2 = e_3 = 4.7 \times 10^{-13}$ (尔格/粒子) b	$u^0 = 1.4 \times 10^5$ (厘米/秒) g
$h\nu = e_3 - e_1 = 1.9 \times 10^{-13}$	$T^0 = 300°K$
$m = 2.92 \times 10^{-23}$ (克) c	$K_{10}^0 = 5.68 \times 10^4$ (秒⁻¹) h
$c_p = 1.4 \times 10^7$ d (尔格/克, °K)	$K_{32}^0 = 2.77 \times 10^4$
$\sum_{i=0}^{3} N_i/N = 0.5$	$K_{21}^0 = 1.02 \times 10^4$
$(K_{32}, K_{21}, K_{10}) = (K_{32}^0, K_{21}^0, K_{10}^0)\frac{\rho}{\rho^0}\sqrt{\frac{T}{T^0}}$	$K_{31}^0 = 4.02 \times 10^5$
$K_{23} = K_{32}\exp\left(-\frac{300}{T}\right)$	$CO_2/N_2/He = 1/4/5$

KEY: a - constant b - (ergs per particle)
c - (gram) d - (ergs/gram, °K) e - (particles per cubic centimeter) f - (dynes per square centimeter) g - (centimeters per second)
h - (second^{-1}) i - (square centimeter)
j - (centimeter^{-1})

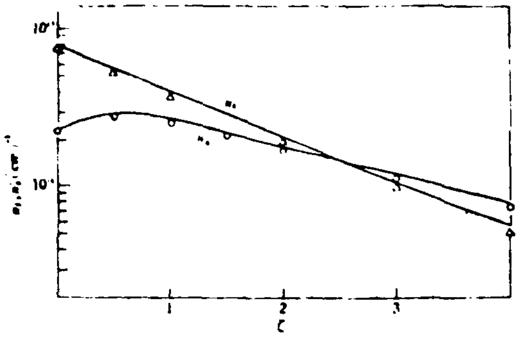

Fig. 3. Variation of n_3 and n_b with ζ
Legend: ——— exact numercal solution
Δ o is the approximate solution in this article
Δ n_3 o n_b

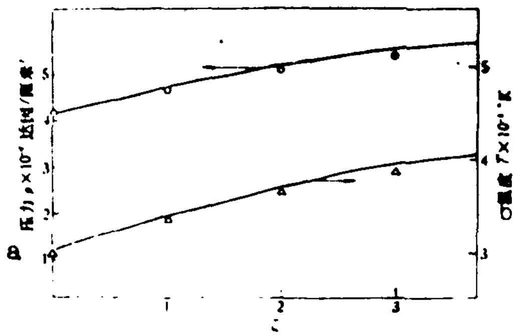

Fig. 4. Variation of pressure and temperature with ζ
Legend: ——— exact numerical solution
o Δ is the approximate solution in this article
o pressure Δ temperature
KEY: a - pressure p x 10^{-4} dynes per square centimeter
b - temperature Tx10^{-2} ^{o}K

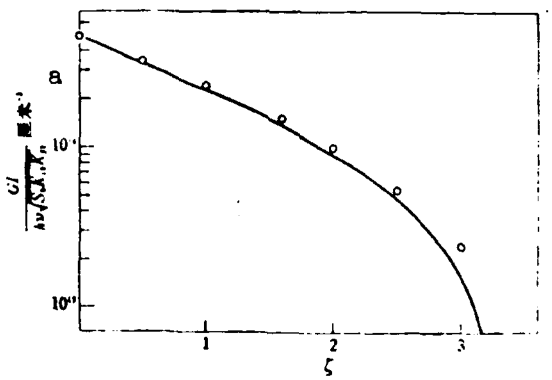

Fig. 5. Variation of power density with ζ
Legend: ——— exact numerical solution; o approximate solution in this article
KEY: a - centimeter^{-3}

After introducing the linear vector [4,5] of the revised pressure effect, the gain coefficient is

$$G = \frac{1}{\pi \Delta \nu_N} \Phi(\xi, \eta, 0)(B_{21}\alpha_2 n_2 - B_{12}\alpha_1 n_1) \tag{6.2}$$

By a derivation that is similar to that in section 4, and by utilizing the condition that gain is equal to loss, the following is derived:

$$G = \frac{G_{0n}\Phi(\xi, \eta, 0)}{1 + \bar{I}_R \Phi(\xi, \eta, 0)} \tag{6.3}$$

This equation and the references [5,6] have the same results, applicable when gain is equal to loss. When high pressure $\eta \gg 1$, Eqs. (6.3) and (5.1) are of the same order of magnitude. In these two theories, G_{0n} and loss G are the same. Therefore, from Eqs. (5.1) and (6.3), we derive

$$\bar{I}_R = \frac{1 + \bar{I}_K}{\Phi(\xi, \eta, \bar{I}_K)} - \frac{1}{\Phi(\xi, \eta, 0)} \tag{6.4}$$

We can see in all the possible values of ξ and η , the intensity $\bar{I}_R$ of RET are greater than the intensity $\bar{I}_K$ in this theory; refer to Fig. 6; refer to Fig. 7 for further explanations. In the figure, by using $\bar{I}$ and ξ as parameters, and given the variation relationship of G/G_{0n} with η , all curves in this theory are situated below the corresponding RET curves; all curves in the two theories are situated below the homogeneous broadening limit curves. This explains the effect of RET on low pressure; that is, the estimation of the effect is insufficient for the inhomogeneous broadening effect. For the situation of the broadening parameter $\eta < 0.2$, it is necessary to adopt the results of the kinematics theory.

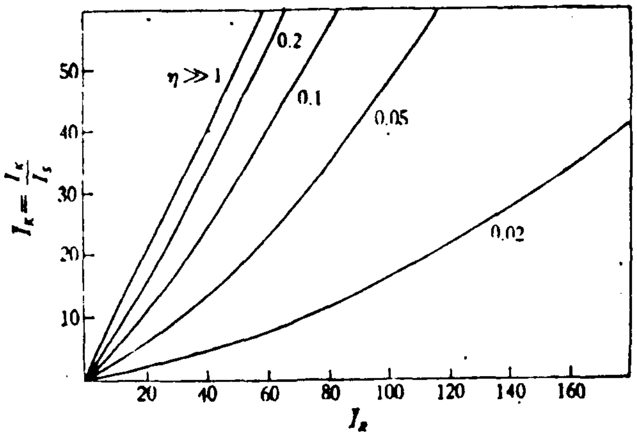

Fig. 6. Relationship between $\bar{I}_K$ and $\bar{I}_R$ (ξ=0)

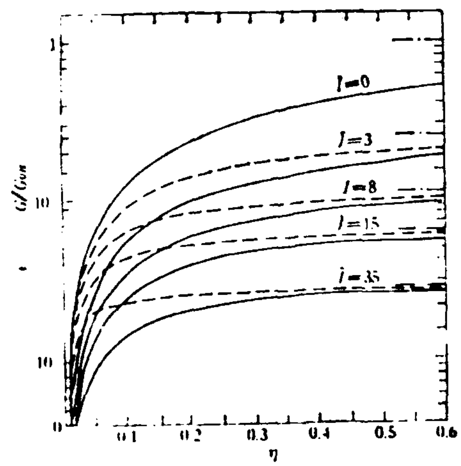

Fig. 7. Variation of G/G_{0n} with η (ξ=0.5)
Legend: ——— this theory
– – – – RET theory
——·—— homogeneous broadening limit

3. Comparison with the gas (not flowing) laser theory: the apparent dependence on ζ of parameters such as f_i^0 is in the form of index $e^{-\lambda_i \zeta}$, such as Eq. (4.16). Therefore, when $\lambda_i \zeta \gg 1$, the relationships between Eq. (4.16) and (5.3) can be simplified as

$$\bar{f}_1^0 = \frac{1}{s_2 k_{10}} \sum_i^3 \gamma_i + \frac{s_0 G_T^0}{1 + (\lambda_1 + \lambda_2)\delta} \left[\frac{(k_{10} - k_{23})\delta}{\sqrt{\mu}} + \frac{1}{s_2} \right]$$

$$\bar{f}_3^0 = \frac{\gamma_3}{k_{32}} + \frac{s_1 k_{23}}{\mu} \sum_i^3 \gamma_i + \frac{s_0 k_{23} G_T^0}{1 + (\lambda_1 + \lambda_2)\delta} \left(\frac{\delta}{\sqrt{\mu}} + \frac{k_{10}}{\mu} \right)$$

$$\bar{J}_s = \frac{\pi \Delta \nu_N}{2} \frac{c h \nu s_2}{B_{21} a_2} \frac{k_{21} k_{32} k_{10} + [k_{21} k_{32} + (k_{32} + k_{23} + k_{31}) k_{10}] \delta \sqrt{\mu}}{s_2 k_{32} k_{10} + (k_{32} + s_1 k_{23} + s_2 k_{10}) \delta \sqrt{\mu}}$$

$$\bar{G}_{0m} \bar{J}_s = h\nu \left(\gamma_2' + \gamma_3' - \frac{s_1 k_{21}}{s_2 k_{10}} \sum_i^3 \gamma_i' \right)$$

(6.5)

In the equation, the first order term of δ is retained, and the second and higher order terms of δ are neglected; (it can be proved that $\delta \ll 1$). When the reflectivity of the mirror does not vary with x, that is, $\delta = 0$. Eq. (6.5) and the corresponding equation (5.1) are just the familiar relationship [11] of the gas (not flowing) laser. It is apparent that the well-known relationship [11] of the gas (not flowing) laser is a special case of this theory when u = 0 or $\lambda_i \zeta \gg 1$. However, it should be noted that Eq. (6.5) is suitable for the case when the gas properties vary with the flow direction. From x = 0 satisfying the relationship (6.5), the gas flows past a distance x_p as

$$x_p \approx \frac{2u}{\lambda_1 \sqrt{\mu}} \approx \frac{4}{\sqrt{s_2 k_{32} k_{10}}} \qquad (6.6)$$

7. Conclusions

Results of the approximate theory in this article are applicable to the entire pressure range; the approximate results

and the results of exact value match quite closely. At high pressure, the results are consistent with the rate equation theory that is generally used. The familiar relationship [11] of the gas (not flowing) laser can also be obtained as a special case of the result of this article. This illustrates that the present treatment of the kinematics theory, the introduction of gain related to molecular velocity, and the corresponding approximate solution method can serve in relatively exactly calculating the macroscopic and microscopic motions of the gas, as well as the interdependent properties of the three, including the radiation field.

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Footnotes:

1. This article was circulated in the two following cases: the Second All-China Fluid Mechanics and the First Asia Fluid Mechanics Conference at Bangalore, India, in December 1980.

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